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A NOVEL APPROXIMATION ALGORITHM FOR THE FORWARD DISPLACEMENT ANALYSIS OF THE 6-DOF PARALLEL MANIPULATOR

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Abstract

In this paper, the forward displacement problem of 6-dof parallel manipulator is transformed into a new equivalent one, and then a novel approximation algorithm with an obvious physical significance is adopted to solve it. The characteristic of the presented algorithm is that, its convergence domain is bigger than that of the general approximation algorithm. Numerical simulation is illustrated to verify the validity and effectiveness of the algorithm. The proposed method is general and can be used for the forward displacement analysis of the parallel manipulator actuated by the revolute joint.

1. Introduction

A parallel manipulator is a closed-loop kinematic chain mechanism, whose end-effector is linked to the base by several independent kinematic chains [24]. Parallel manipulators have received increasingly attention due to their inherent advantages over the conventional serial mechanism

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after the original inventor [1-42]. They have been successfully used in the flight and automotive simulators, robotic end-effector, and the other circumstances like the fast pick-and-place operation.

This forward displacement problem is to obtain the orientation and position of the moving platform with respect to the base plate from the knowledge of joint variables and the geometry of the base and moving plate. Forward displacement analysis is one of the important issues in the development of parallel manipulator. It is usually used for the computer simulation and virtual prototype building. Furthermore, the motion planning and control of the manipulator call for the forward displacement analysis. Efficient solutions of this problem allow the algorithms for the control and motion planning of the manipulator configuration to be improved. The forward displacement problem involves systems of highly non-collinear equations, which lead to be quite difficult. Approaches for solving the problem can be classified into two categories, i.e., analytical method and numerical method. Many researchers have studied it in the analytical method [2, 9-12, 17, 20, 30, 36]. They applied different elimination procedures to eliminate the unknowns for a set of constraint equations and to form one polynomial equation with single variable. So, the major task of these kinds of methods is to solve the polynomial equation with one variable in the closed form. The elimination procedures are usually very complicated and they often introduce unnecessary complex roots. In addition, even the polynomial equation derived successfully, it is a challenging and time-consuming task to search out all the possible solution.

Numerical methods were adopted by many researchers to solve the forward displacement problem. Newton-Raphson method was used to solve the six kinematic equations with six Cartesian variables [16, 31]. A modified Powell method was developed to solve the six kinematic equations for a 6-6 Stewart platform with semi-regular hexagonal geometry [3]. Since the initial vector does not have to be very close to the solution vector and the explicit evaluation of the derivative matrix is unnecessary, the modified Powell method is superior to the Newton-

Raphson, Broyden, and Marquardt methods with respect to the number of function evaluations and computational time. The direct kinematic problem was transformed to the problem of global minimization of sum of squares objective function and solved by using the general optimization method. The problem was formulated in the domain of complex numbers [29]. A continuation method was adopted to solve the direct position kinematics of a fully general Stewart platform [27]. Tracking 960 paths from the start system, he was able to find 40 distinct solutions in the complex domain, which suggests that the least upper bound of the number of configurations of the Stewart platform may be 40. An exhaustive numerical approach algorithm was presented in the Cartesian variables [14]. They used a mono-dimensional-search rather than a six-dimensional search in the Cartesian variables. A robust estimator design was proposed to solve the forward kinematics [33]. In addition, several other methods such as neural network and genetic algorithm were also used to solve the problem [1, 8, 28, 38, 39]. Interval analysis was adopted to solve all the possible poses of the platform for given joint coordinates [4, 23]. A successive approximation algorithm with much small convergence domain was developed for the direct position analysis of the parallel manipulator [41, 42]. It should be pointed that, the numerical approach usually can be used to find the solutions closed to the initial estimate by using root algorithms or optimization techniques [16].

When adopting the numerical method to the forward displacement analysis of the parallel manipulator actuated by the revolute joint, the procedure of the solution often be interrupted or the result has no physical significance due to the extraction of the square root. This paper presents a novel numerical method for the forward displacement analysis of the 6-dof parallel manipulator. The problem has been transformed into a new equivalent one, and then the novel approximation algorithm with an obvious physical significance is adopted to solve it. Simulation is given to illustrate the validity and effectiveness of the algorithm. The proposed method is general and can be used for the forward displacement analysis of the parallel manipulator actuated by the revolute joint.

2. System Description

The schematic diagram of the 6-dof parallel manipulator is shown in Figures 1 and 2. It is a 6-PSS parallel manipulator, which consists of a platform and six sliders. In each kinematic chain, the platform and the slider are connected via spherical ball bearing joints by a strut of fixed length. Each slider is driven by a DC motor via a linear ball screw. The lead screw of B_1, B_2, and B_3 are vertical to the ground. The lead screw of B_4, B_5, and B_6 are parallel with the ground and are orthogonal to lead screw of B_1, B_2, and B_3.

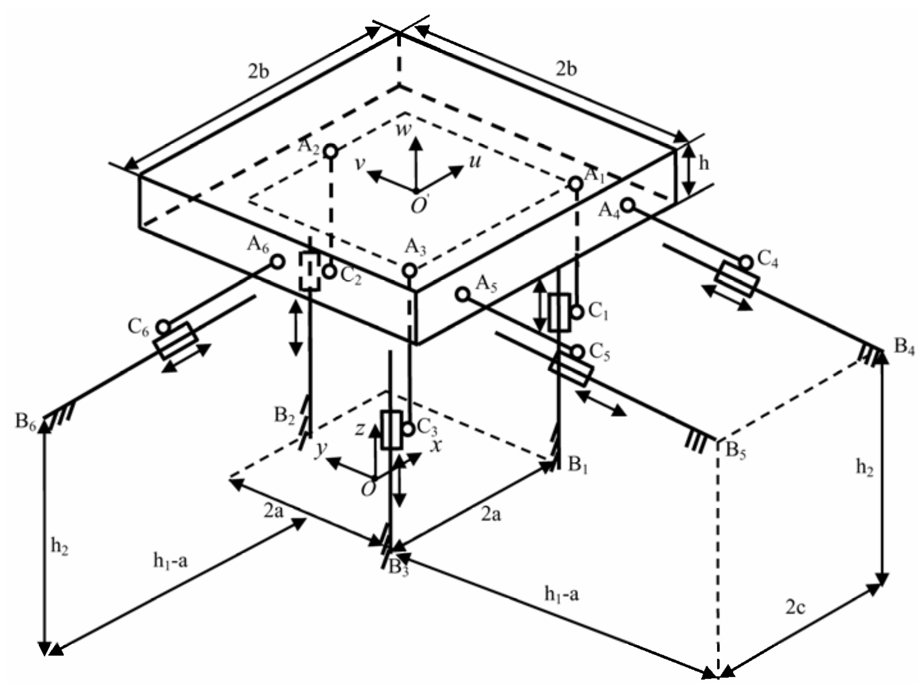

Figure 1. Schematic diagram of the 6-dof parallel manipulator.

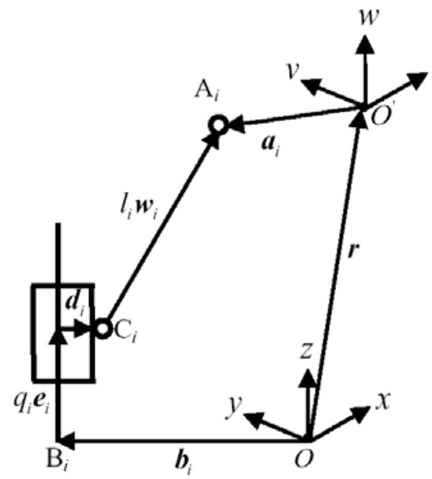

Figure 2. Vector diagram of a PSS kinematic chain.

For the purpose of analysis, the following coordinate systems are defined: The coordinate system $O - xyz$ is attached to the fixed base and another moving coordinate frame $O' - uvw$ is located at the center of mass of the moving platform. The pose of the moving platform can be described by a position vector $\boldsymbol{r}$, and a rotation matrix ${}^{o}\boldsymbol{R}_{o'}$. Let the rotation matrix be defined by the roll, pitch, and yaw angles, namely, a rotation of ϕ_x about the fixed x axis, followed by a rotation of ϕ_y about the fix y axis, and a rotation of ϕ_z about the fix z axis. Thus, the rotation matrix is

$$
\begin{aligned}
{}^{o}\boldsymbol{R}_{o'} &= \mathrm{Rot}(z, \phi_z)\mathrm{Rot}(y, \phi_y)\mathrm{Rot}(x, \phi_x) \\
&= \begin{bmatrix} c\phi_z c\phi_y & c\phi_z s\phi_y s\phi_x - s\phi_z c\phi_x & c\phi_z s\phi_y c\phi_x + s\phi_z s\phi_x \\ s\phi_z c\phi_y & s\phi_z s\phi_y s\phi_x + c\phi_z c\phi_x & s\phi_z s\phi_y c\phi_x - c\phi_z s\phi_x \\ -s\phi_y & c\phi_y s\phi_x & c\phi_y c\phi_x \end{bmatrix}, \quad (1)
\end{aligned}
$$

where $c\phi = \cos\phi$, $s\phi = \sin\phi$.

3. Forward Displacement Analysis

As shown in Figure 2, the closed-loop position equation associated with the i-th kinematic chain can be written as

$$\boldsymbol{r} + \boldsymbol{a}_i = l_i \boldsymbol{w}_i + \boldsymbol{b}_i + \boldsymbol{d}_i + q_i \boldsymbol{e}_i, \tag{2}$$

where $\boldsymbol{r}$, q_i, $\boldsymbol{e}_i$, $\boldsymbol{w}_i$, $\boldsymbol{a}_i$, $\boldsymbol{b}_i$, and $\boldsymbol{d}_i$ denote the vector $\boldsymbol{OO'}$, the joint variable, the unit vector along the lead screw, the unit vector along strut C_iA_i, the vector $\boldsymbol{O'A}_i$, the vector $\boldsymbol{OB}_i$, and the vector from the lead screw to the center point of the joint C_i, respectively. Taking the derivative of Equation (2) with respect to time yields

$$\dot{q}_i \boldsymbol{e}_i + \omega_i \times l_i \boldsymbol{w}_i = \nu + \omega \times \boldsymbol{a}_i, \tag{3}$$

where ω_i and ν denote the angular velocity of the strut C_iA_i and the linear velocity of the moving platform.

Taking the dot product of both sides of Equation (3) with $\boldsymbol{w}_i$ yields

$$\dot{q}_i = \begin{bmatrix} \dfrac{\boldsymbol{w}_i^T}{\boldsymbol{w}_i^T \boldsymbol{e}_i} & \dfrac{(\boldsymbol{a}_i \times \boldsymbol{w}_i)^T}{\boldsymbol{w}_i^T \boldsymbol{e}_i} \end{bmatrix} \begin{bmatrix} \nu \\ \omega \end{bmatrix}. \tag{4}$$

Rewriting Equation (4) in the matrix form yields

$$\dot{\boldsymbol{q}} = \boldsymbol{J}_q^{-1} \boldsymbol{J}_x \dot{\boldsymbol{X}} = \boldsymbol{J}\dot{\boldsymbol{X}}, \tag{5}$$

where

$$\dot{\boldsymbol{q}} = [\dot{q}_1 \quad \dot{q}_2 \quad \dot{q}_3 \quad \dot{q}_4 \quad \dot{q}_5 \quad \dot{q}_6]^T; \tag{6}$$

$$\dot{\boldsymbol{X}} = \begin{bmatrix} \nu \\ \omega \end{bmatrix}; \tag{7}$$

$$\boldsymbol{J}_q = \mathrm{diag}(\boldsymbol{w}_1^T \boldsymbol{e}_1 \;\; \boldsymbol{w}_2^T \boldsymbol{e}_2 \;\; \boldsymbol{w}_3^T \boldsymbol{e}_3 \;\; \boldsymbol{w}_4^T \boldsymbol{e}_4 \;\; \boldsymbol{w}_5^T \boldsymbol{e}_5 \;\; \boldsymbol{w}_6^T \boldsymbol{e}_6); \tag{8}$$

$$\boldsymbol{J}_x = \begin{bmatrix} \boldsymbol{w}_1 & \boldsymbol{w}_2 & \boldsymbol{w}_3 & \boldsymbol{w}_4 & \boldsymbol{w}_5 & \boldsymbol{w}_6 \\ \boldsymbol{a}_1 \times \boldsymbol{w}_1 & \boldsymbol{a}_2 \times \boldsymbol{w}_2 & \boldsymbol{a}_3 \times \boldsymbol{w}_3 & \boldsymbol{a}_4 \times \boldsymbol{w}_4 & \boldsymbol{a}_5 \times \boldsymbol{w}_5 & \boldsymbol{a}_6 \times \boldsymbol{w}_6 \end{bmatrix}^T, \tag{9}$$

where $\boldsymbol{J}$ is the *Jacobian matrix*, which maps the velocity vector $\dot{\boldsymbol{X}}$ into the joint velocity vector $\dot{\boldsymbol{q}}$.

3.1. General approximation algorithm

Suppose the desired approximation precision is e. Let $\boldsymbol{X}_0$ be the initial estimated pose of the moving platform. The direct displacement problem can be solved as the following general approximation algorithm:

Step 1. Calculate the joint variables of the six kinematic chains corresponding to the initial estimated pose $\boldsymbol{X}_0$.

The joint variables q_{0i} corresponding to the instantaneous configuration determined by the $\boldsymbol{X}_0$ can be calculated as:

$$\boldsymbol{q}_{i0} = \sqrt{(\boldsymbol{r}_0 + \boldsymbol{a}_{i0} - l_i\boldsymbol{w}_{i0} - \boldsymbol{b}_i - \boldsymbol{d}_i)^T(\boldsymbol{r}_0 + \boldsymbol{a}_{i0} - l_i\boldsymbol{w}_{i0} - \boldsymbol{b}_i - \boldsymbol{d}_i)}, \tag{10}$$

where

$$\boldsymbol{a}_{i0} = {}^{o}\boldsymbol{R}_{o'0}\,{}^{o'}\boldsymbol{a}_{i0}, \tag{11}$$

$${}^{o}\boldsymbol{R}_{o'0} = \mathrm{Rot}(z, \boldsymbol{X}_0(6, 1))\mathrm{Rot}(y, \boldsymbol{X}_0(5, 1))\mathrm{Rot}(x, \boldsymbol{X}_0(4, 1)). \tag{12}$$

Step 2. Calculate the approximation step that the moving platform need to move.

The Jacobian matrix of the instantaneous configuration is achieved through Equations (5), (6), (7), (8), and (9). The difference between the six input joint variables and the instantaneous six joint variables is

$$\Delta\boldsymbol{q} = [q_1 - q_{01}\ \ q_2 - q_{02}\ \ q_3 - q_{03}\ \ q_4 - q_{04}\ \ q_5 - q_{05}\ \ q_6 - q_{06}]^T. \tag{13}$$

In order to approximate the desired configuration, the pose vector difference, which the moving platform should move is

$$\Delta\boldsymbol{X} = \boldsymbol{J}^{-1}\Delta\boldsymbol{q}, \tag{14}$$

where $\boldsymbol{J}$ is the Jacobian matrix corresponding to the instantaneous configuration of the parallel manipulator.

Step 3. Calculate the new pose vector of the moving platform $\boldsymbol{X}_1$, which represent the new instantaneous configuration.

$$\boldsymbol{X}_1 = \boldsymbol{X}_0 + \lambda\Delta\boldsymbol{X} = \boldsymbol{X} + \lambda\boldsymbol{J}^{-1}\Delta\boldsymbol{q}, \tag{15}$$

where $\lambda \in (0, \gamma)$ is the coefficient and γ can be bigger than one.

Step 4. Calculate the new six joint variables q_{1i} corresponding to the new instantaneous configuration is determined by the pose vector $\boldsymbol{X}_1$.

$$q_{i1} = \sqrt{(\boldsymbol{r}_1 + \boldsymbol{a}_{i1} - l_i\boldsymbol{w}_{i1} - \boldsymbol{b}_i - \boldsymbol{d}_i)^T(\boldsymbol{r}_1 + \boldsymbol{a}_{i1} - l_i\boldsymbol{w}_{i1} - \boldsymbol{b}_i - \boldsymbol{d}_i)}, \tag{16}$$

where

$$\boldsymbol{a}_{i1} = {}^o\boldsymbol{R}_{o'1}{}^{o'}\boldsymbol{a}_{i1}, \tag{17}$$

$${}^o\boldsymbol{R}_{o'1} = \mathrm{Rot}(z, \Delta\boldsymbol{X}(6, 1))\mathrm{Rot}(y, \Delta\boldsymbol{X}(5, 1))\mathrm{Rot}(x, \Delta\boldsymbol{X}(4, 1))^o\boldsymbol{R}_{o'0}. \tag{18}$$

So, joint variables corresponding to the instantaneous configuration are:

$$q_{01} = A_{1z} - \sqrt{l_1^2 - (A_{1x} - a)^2 - (A_{1y} + a - d_1)^2}; \tag{19a}$$

$$q_{02} = A_{2z} - \sqrt{l_2^2 - (A_{2x})^2 - (A_{2y} - a - d_2)^2}; \tag{19b}$$

$$q_{03} = A_{3z} - \sqrt{l_3^2 - (A_{3x} + a)^2 - (A_{3y} + a - d_3)^2}; \tag{19c}$$

$$q_{04} = A_{4y} + h_1 - \sqrt{l_4^2 - (A_{4x} - c)^2 - (A_{4z} - h_2 - d_4)^2}; \tag{19d}$$

$$q_{05} = A_{5y} + h_1 - \sqrt{l_5^2 - (A_{5x} + c)^2 - (A_{5z} - h_2 - d_5)^2}; \tag{19e}$$

$$q_{06} = A_{6x} + h_1 - \sqrt{l_6^2 - (A_{6y})^2 - (A_{6z} - h_2 - d_6)^2}, \tag{19f}$$

where A_{ix}, A_{iy}, and A_{iz} are the coordinates of the point A_i in the coordinate system $O - xyz$.

Step 5. If the following condition

$$\|\boldsymbol{q} - \boldsymbol{q}_1\| \le e \tag{20}$$

is satisfied, stop the approximation. The instantaneous configuration is the desired one. $\|\bullet\|$ is the norm of the vector, which can be chosen 1-norm, 2-norm, and ∞- norm. We select 2-norm in this paper.

If the above condition is not satisfied, let

$$\boldsymbol{X}_0 = \boldsymbol{X}_1, \tag{21}$$

$$^{o}\boldsymbol{R}_{o'0} = {}^{o}\boldsymbol{R}_{o'1}. \tag{22}$$

Go to the Step 2 to continue the approximation.

The above general approximation algorithm has been used successfully for the direct displacement analysis of the in-parallel manipulator. However, it must be pointed that it often failures, if the initial joint variables q_{0i} are not given correctly, when used for the direct displacement analysis of the parallel manipulator actuated by the revolute joint. Due to Equation (19), the result of the joint variable is often a complex number. There is no physical significance for a complex joint variable. So, approximation of the solution to the displacement problem often be interrupted or the result has no physical significance. In order to avoid this phenomenon, the direct displacement problem of the 6-dof parallel manipulator can also be transformed into a new equivalent one.

3.2. Novel approximation algorithm

Suppose that the strut C_iA_i is extensible while the slider is fixed, to determine the position of the point A_i, when the length of the struts is given. This is the main idea of the equivalent problem. On this postulated condition, taking the derivative of the Equation (2) with respect to time yields

$$\dot{l}_i\boldsymbol{w}_i + \omega_i \times l_i\boldsymbol{w}_i = \nu + \omega \times \boldsymbol{a}_i. \tag{23}$$

Taking the dot product of both sides of Equation (23) with $\boldsymbol{w}_i$, then yields

$$\dot{l}_i = [\boldsymbol{w}_i^T (\boldsymbol{a}_i \times \boldsymbol{w}_i)^T] \begin{bmatrix} \nu \\ \omega \end{bmatrix}. \tag{24}$$

Rewriting Equation (24) in the matrix form yields

$$\dot{\boldsymbol{L}} = \boldsymbol{J}_L \dot{\boldsymbol{X}}, \tag{25}$$

where

$$\dot{\boldsymbol{L}} = [\dot{l}_1 \quad \dot{l}_2 \quad \dot{l}_3 \quad \dot{l}_4 \quad \dot{l}_5 \quad \dot{l}_6]^T; \tag{26}$$

$$\dot{\boldsymbol{X}} = \begin{bmatrix} \nu \\ \omega \end{bmatrix}; \tag{27}$$

$$\boldsymbol{J}_L = \begin{bmatrix} \boldsymbol{w}_1 & \boldsymbol{w}_2 & \boldsymbol{w}_3 & \boldsymbol{w}_4 & \boldsymbol{w}_5 & \boldsymbol{w}_6 \\ \boldsymbol{a}_1 \times \boldsymbol{w}_1 & \boldsymbol{a}_2 \times \boldsymbol{w}_2 & \boldsymbol{a}_3 \times \boldsymbol{w}_3 & \boldsymbol{a}_4 \times \boldsymbol{w}_4 & \boldsymbol{a}_5 \times \boldsymbol{w}_5 & \boldsymbol{a}_6 \times \boldsymbol{w}_6 \end{bmatrix}^T. \tag{28}$$

The equivalent direct kinematics problem is solved in the following:

Step 1. Calculate the joint variables of the six kinematic chains corresponding to the initial estimated pose $\boldsymbol{X}_0$.

The initial length of the supposed extensive link corresponding to the instantaneous configuration determined by the $\boldsymbol{X}_0$ can be calculated as:

$$l_{0i} = \sqrt{(\boldsymbol{r}_0 + \boldsymbol{a}_{i0} - \boldsymbol{b}_i - \boldsymbol{d}_i - q_i \boldsymbol{e}_i)^T (\boldsymbol{r}_0 + \boldsymbol{a}_{i0} - \boldsymbol{b}_i - \boldsymbol{d}_i - q_i \boldsymbol{e}_i)}, \tag{29}$$

where

$$\boldsymbol{a}_{i0} = {}^{o}\boldsymbol{R}_{o'0} \, {}^{o'}\boldsymbol{a}_{i0}, \tag{30}$$

$${}^{o}\boldsymbol{R}_{o'0} = \mathrm{Rot}(z, \boldsymbol{X}_0(6, 1))\mathrm{Rot}(y, \boldsymbol{X}_0(5, 1))\mathrm{Rot}(x, \boldsymbol{X}_0(4, 1)). \tag{31}$$

Step 2. Calculate the approximation step that the moving platform need to move. The difference between the six input legs length and the supposed instantaneous six legs length is

$$\Delta \boldsymbol{L} = [l_1 - l_{01} \;\; l_2 - l_{02} \;\; l_3 - l_{03} \;\; l_4 - l_{04} \;\; l_5 - l_{05} \;\; l_6 - l_{06}]^T. \tag{32}$$

In order to approximate the desired configuration, the pose vector difference, which the moving platform should move is

$$\Delta \boldsymbol{X} = \boldsymbol{J}_L^{-1} \Delta \boldsymbol{L}. \tag{33}$$

Step 3. Calculate the new pose vector of the moving platform $\boldsymbol{X}_1$, which represent the new instantaneous configuration.

$$\boldsymbol{X}_1 = \boldsymbol{X}_0 + \lambda \Delta \boldsymbol{X} = \boldsymbol{X} + \lambda \boldsymbol{J}^{-1} \Delta \boldsymbol{q}, \tag{34}$$

where $\lambda \in (0, \gamma)$ is the coefficient and γ can be bigger than one.

Step 4. Calculate the new length of the supposed link corresponding to the new instantaneous configuration is determined by the pose vector $\boldsymbol{X}_1$.

$$l_{1i} = \sqrt{(\boldsymbol{r}_1 + \boldsymbol{a}_{i1} - \boldsymbol{b}_i - \boldsymbol{d}_i - q_i \boldsymbol{e}_i)^T (\boldsymbol{r}_1 + \boldsymbol{a}_{i1} - \boldsymbol{b}_i - \boldsymbol{d}_i - q_i \boldsymbol{e}_i)}, \tag{35}$$

$$\boldsymbol{L}_1 = [l_{11} \quad l_{12} \quad l_{13} \quad l_{14} \quad l_{15} \quad l_{16}]^T, \tag{36}$$

where

$$\boldsymbol{a}_{i1} = {}^{o}\boldsymbol{R}_{o'1} {}^{o'}\boldsymbol{a}_{i1}, \tag{37}$$

$${}^{o}\boldsymbol{R}_{o'1} = \mathrm{Rot}(z, \Delta \boldsymbol{X}(6, 1)) \mathrm{Rot}(y, \Delta \boldsymbol{X}(5, 1)) \mathrm{Rot}(x, \Delta \boldsymbol{X}(4, 1)) {}^{o}\boldsymbol{R}_{o'0}. \tag{38}$$

Step 5. If the following condition

$$\| \boldsymbol{L} - \boldsymbol{L}_1 \| \le e \tag{39}$$

is satisfied, stop the approximation. The instantaneous configuration is the desired one.

There is no extraction of square root of quadratic equation like Equation (19) in this approximation. So, its convergence domain is bigger than that of the general iterative method. This is the characteristic of the novel approximation algorithm for the forward kinematics of the 6-dof parallel manipulator.

4. Numerical Simulation

Adopting the presented method, we solve the direct kinematic problem of the 6-dof parallel manipulator. The structure parameters and the input joint variables are given in Table 1 through Table 5.

Table 1. The parameters of the base platform (m)

	1	2	3	4	5	6
x_{Bi}	0.300000	0.000000	– 0.300000	0.300000	– 0.300000	– 1.607000
y_{Bi}	– 0.300000	0.300000	– 0.300000	– 1.607000	– 1.607000	0.000000
z_{Bi}	0.000000	0.000000	0.000000	1.437000	1.437000	1.437000

Table 2. The parameters of the moving platform, which are measured in the coordinate frame $O' - uvw$ (m)

	1	2	3	4	5	6
x_{Ai}	0.300000	0.000000	– 0.300000	0.300000	– 0.300000	– 0.581000
y_{Ai}	– 0.300000	0.300000	– 0.300000	– 0.581000	– 0.581000	0.000000
z_{Ai}	– 0.266000	– 0.266000	– 0.266000	– 0.037500	– 0.037500	– 0.037500

Table 3. The length of the strut C_iA_i (m)

	1	2	3	4	5	6
l_i	0.382000	0.362000	0.382000	0.382000	0.382000	0.362000

Table 4. The input joint variable (m)

	1	2	3	4	5	6
q_i	0.800000	0.900000	1.000000	0.900000	0.800000	0.700000

Table 5. The position of the spherical ball bearing joint C_i (m)

	1	2	3	4	5	6
x_{Ci}	0.300000	0.000000	– 0.300000	0.300000	– 0.300000	– 0.907000
y_{Ci}	– 0.056000	0.544000	– 0.056000	– 0.707000	– 0.807000	0.000000
z_{Ci}	0.866000	0.966000	1.066000	1.681000	1.681000	1.681000

We choose the approximation precision $e = 10^{-6}$ and the coefficient $\lambda = 1.$ The results are shown in Table 6, where $\boldsymbol{L}_i$ is the length vector of the supposed extensive links corresponding to the instantaneous configuration in the approximation process. Other parameter used in the simulation is given as $d_i = 0.244$m.

The number of steps of the novel approximation algorithm for the forward displacement analysis of the 6-dof parallel manipulator is only six. The result of the numerical simulation shows that the algorithm is very effective.

Table 6. The process and the last result by virtue of novel approximation algorithm (SI)

$\boldsymbol{X}$	$\boldsymbol{X}_0$	$\boldsymbol{X}_1$	$\boldsymbol{X}_2$	$\boldsymbol{X}_3$	$\boldsymbol{X}_4$	$\boldsymbol{X}_5$	$\boldsymbol{X}_6$	$\boldsymbol{X}$
x	0.000000	0.050192	0.058464	0.002393	– 0.011837	– 0.014314	– 0.014526	– 0.014528
y	0.000000	0.375233	0.288196	0.185060	0.174403	0.169775	0.169467	0.169463
z	1.860000	1.676052	1.631098	1.585370	1.562882	1.559887	1.559677	1.559674
ϕ_x	0.000000	– 0.02390	– 0.138131	– 0.051880	– 0.065038	– 0.062074	– 0.061693	– 0.061688
ϕ_y	0.000000	0.171707	0.298800	0.344658	0.334854	0.338818	0.339370	0.339376
ϕ_z	0.000000	0.546696	0.006223	0.111493	0.062776	0.055135	0.054050	0.054038
$\boldsymbol{L}$	$\boldsymbol{L}_0$	$\boldsymbol{L}_1$	$\boldsymbol{L}_2$	$\boldsymbol{L}_3$	$\boldsymbol{L}_4$	$\boldsymbol{L}_5$	$\boldsymbol{L}_6$	$\boldsymbol{L}$
l_1	0.767802	0.599534	0.464975	0.391299	0.383871	0.382121	0.382001	0.382000
l_2	0.673736	0.466945	0.375294	0.385302	0.362474	0.362017	0.362000	0.362000
l_3	0.581653	0.461736	0.441252	0.402007	0.383073	0.382049	0.382001	0.382000
l_4	0.189468	0.800180	0.432031	0.402997	0.384816	0.382198	0.382003	0.382000
l_5	0.266643	0.662022	0.526383	0.390975	0.385352	0.382100	0.382001	0.382000
l_6	0.355385	0.472466	0.494886	0.379373	0.364143	0.362019	0.362001	0.362000

With the same initial configuration, Table 7 shows the process and the results of the general approximation algorithm. It can be seen that the approximation process may also be unsuccessful even, when the initial $\boldsymbol{X}_0$ is not out of the workspace of the 6-dof parallel manipulator. Since, there is no extraction of square root of quadratic equation like Equation (19) in the procedure of the general approximation, its convergence domain is bigger than that of the general approximation algorithm. This is the characteristic of the presented approximation algorithm for the forward displacement analysis of the 6-dof parallel manipulator.

Table 7. The process and the last result by virtue of general approximation algorithm (SI)

$\boldsymbol{X}$	$\boldsymbol{X}_0$	$\boldsymbol{X}_1$	$\boldsymbol{X}_2$	$\boldsymbol{X}_{i\ i=3,4\cdots,7}$	$\boldsymbol{X}$
x	0.000000	0.055106	0.063684	Complex Numbers	Complex Numbers
y	0.000000	0.363990	– 0.010810		
z	1.860000	1.647534	1.632797		
ϕ_x	0.000000	– 0.034076	– 0.021449		
ϕ_y	0.000000	0.206418	0.086810		
ϕ_z	0.000000	0.506757	0.340664		

$\boldsymbol{q}$	$\boldsymbol{q}_0$	$\boldsymbol{q}_1$	$\boldsymbol{q}_{i\ i=2,3\cdots7}$	$\boldsymbol{q}$
$\boldsymbol{q}_1$	1.234082	1.022485	Complex Numbers	Complex Numbers
$\boldsymbol{q}_2$	1.260590	0.978909		
$\boldsymbol{q}_3$	1.234082	1.069405		
$\boldsymbol{q}_4$	0.671174	1.382534		
$\boldsymbol{q}_5$	0.671174	1.261432		
$\boldsymbol{q}_6$	0.692801	0.809904		

5. Conclusion

A novel numerical method for the forward displacement analysis of the 6-dof parallel manipulator is presented in this paper. The problem has been transformed into a new equivalent one, and then the novel approximation algorithm with an obvious physical significance is adopted to solve it. Compared with the general numerical method, there is no extraction of square root of quadratic equation in the iteration procedure. So, its convergence domain is bigger than that of the general iterative method. Simulation is given to illustrate the validity and effectiveness of the algorithm. The proposed method is general and can be used for the forward displacement analysis of the parallel manipulator actuated by the revolute joint.

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